P011 Imaging without a velocity model by path-summation approach: this time in depth

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Abstract
In this paper we generalize a method, proposed for model independent time imaging, to calculate seismic images in depth without performing velocity analysis or using a priory knowledge of the exact subsurface model. By analogy to the use of Feynman path integrals in waveform modeling, we develop an algorithm for depth imaging by summation of elementary signals over a sample of all possible paths between source and receiver. The constructive and destructive interference of these signals, produces an image of the subsurface.

Introduction
It is commonly accepted that achieving correct seismic imaging in depth requires accurate and detailed information about the subsurface velocity model. The conventional imaging techniques usually require velocity estimation procedures which are both time consuming and subjective. Although there are many situations where automatic picking is useful (Fomel, 2003), there are cases where picking one value is not sufficient to determine a unique velocity. Complicated wave propagation effects can often cause serious problems for the assumptions behind most velocity analysis techniques. Another serious objection to velocity estimation/picking is related to a concept of stochastic random nature of subsurface velocity, where only a probabilistic density function (PDF) of velocity can be considered. In this case, imaging with only one ("exact") velocity becomes inaccurate and not practical (Jedlicka, 1996).
Weglein et al. (2000) developed an inverse scattering approach that may provide a subsurface image without having a detailed velocity information. Landa (2004) proposed another model independent imaging algorithm which is based on a heuristic development using "path summation" technique, to obtain the subsurface image without specifying an accurate subsurface model. Path-summation method was previously utilised for obtaining an approximate waveform solutions to the scalar wave equation (Schlottmann, 1999, Lomax, 1999).
The path-summation method constructs an approximate wavefield by summation over the contributions of elementary signals propagated along a representative sample of all possible paths between the source and observation points. The path-integral approach does not rely on the representation of seismic signal by a single event travelling along a specific Fermat's ray. Instead it represents the seismic wave by dense sampling of a large volume between the source and the observation point. Many paths, other than Fermat's path, are defined in the space domain. The contribution from each path varies in phase, according to the Lagrangian of the system.
By analogy to the use of Feynman's path integrals in waveform modelling, Landa (2004) suggested to obtain the subsurface seismic image by such a summation process. In Landa's implementation, the velocity model was unknown and summation trajectories were defined in the data (time) domain rather than in the model (depth) domain. For zero-offset approximation (stack) the path-summation imaging consists of summing prestack seismic data along all possible stacking "hyperbolae" instead of only along a subset, corresponding to the highest coherency criteria (e. g. semblance) in the conventional zero-offset imaging. In the case of time migration (PSTM), the path-summation imaging consists of a summation of elementary signals over all possible diffraction curves instead of only along a subset, corresponding to the chosen migration velocity. The constructive and destructive interference of the elementary signals contributed by each path/trajectory produces an image that converges towards the correct one which is obtained by stack/migration procedure using the correct velocity.
In the following, a heuristic approach which leads to an approximate zero-offset path-summation solution is presented. Next, time and depth migrations without precise knowledge of velocity field are considered as a summation of elementary signals over all possible Green's functions (diffraction curves) instead of only along a subset, corresponding to the chosen migration velocity. The path summation procedure for time/depth imaging will be demonstrated on synthetic and real data examples.

Path-summation zero-offset approximation
Improving the quality of stacked sections remains the focus of intensive research. A lot of effort has been directed towards improving the accuracy of the normal moveout (NMO) correction. Let us represent a stack S for zero-offset time t_0 and location x_0 in the form:

$$S(t_0, x_0) = \sum_x U(t_0 + \tau(\alpha, x, x_0), x) \qquad (1)$$

where $U(t,x)$ is the recorded input data at the CMP location x_0, τ is the time-summation path/trajectory and α is the summation path parameter. x is summed over the measurement aperture.
For a simple CMP stack

$$\tau = \sqrt{t_0^2 + x^2 / v_{st}^2} \qquad (2)$$

where the summation parameter α in equation (1) is equal to a stacking velocity and the integration is done over a CMP gather $U(t,x)$. Usually the stacking velocity v_{st} is estimated by velocity analysis maximising a coherency measure calculated along different traveltimes defined by expression (2).
Let us see what will happen if the data would have been stacked over all possible time trajectories τ. Here we introduce a heuristic development using "path summation" method of quantum mechanics (Feynman and Hibbs, 1965). In Feynman's path integral approach, a particle does not have just a single history, as it would in a classical theory. Instead, it is supposed to follow every possible path in space-time, where each of these histories (trajectories) is associated to the size of a wave and its phase. Based on this principle, a method for approximate solution for waveforms calculation was introduces by Lomax (1999). For a known model, the solution is obtained by summing of signals over a representative sample of all possible space paths between a source and observation point. In analogy to the use of path integrals for seismic modelling, a heuristic construction, based on the path summation idea for imaging without knowing or estimating the model, is introduced. Instead of considering ray paths/trajectories in the spatial domain, all possible time trajectories τ are used for the image construction. In this case equation (1) can be written in the following way:

$$S(t_0, x_0) = \sum_\alpha w(\alpha) \sum_x U(t_0 + \tau(\alpha, x, x_0), x) \qquad (3)$$

where τ now represents all the possible time trajectories, $w(\alpha)$ stands for the weighting factor and α is summed over all possible values. To approximate the CMP zero-offset section the data are stacked along time trajectories corresponding to different stacking velocities.
The weighting function $w(\alpha)$ in this case can be taken as a semblance calculated for each stacking trajectory. The result of the summation approximates a zero-offset trace. It is practically equivalent to the conventionally stacked trace when summation is performed along the time curve corresponding to the true stacking velocity.
The ability of the path-summation approach to resolve conflicting dip situation without using DMO was also presented by Landa (2004). It was demonstrated that the path-summation is an attractive zero-offset imaging method. It possesses several advantages compared to conventional stacking methods, namely, it is automatic, fast and does not require velocity picking. It also incorporates properties of DMO allowing to stack events with different stacking trajectories at the same zero offset time.

Path-summation migration

Migration can be as well considered as a stacking procedure. Migration velocity analysis can become non trivial and time consuming part of prestack migration, particularly PSDM. Let us consider the diffraction stack V for a subsurface location $\vec{x}$ in the form:

$$V(\vec{x}) \sim \int d\xi \int dt U(t,\xi) \delta(t - t_d(\xi, \alpha, \vec{x})) \qquad (4)$$

where $\vec{x}$ corresponds to (t_0, x) for time migration and to (z, x) for depth migration, $U(t,\xi)$ is the recorded input seismic data for an arbitrary source-receiver configuration parameterised by the vector ξ, α is the summation path parameter (migration velocity), and t_d is the summation path over the diffraction travel time curve (the kinematic part of the Green's function) corresponding to this migration velocity. ξ is integrated over the measurement aperture, and the time t over the relevant time interval.
Following the path-summation idea introduced above, let us consider a set of possible time trajectories t_d corresponding to a set of possible velocity models α, and let us use these trajectories for migration image construction. In this case equation (4) can be written in the following way:

$$V(\vec{x}) \sim \int d\alpha w(\alpha) \int d\xi \int dt U(t,\xi) \delta(t - t_d(\xi, \alpha, \vec{x})) \qquad (5)$$

where t_d represents all possible time trajectories, $w(\alpha)$ stands for the weighting factor and α is integrated over all possible values of migration velocity.

Figure 1 schematically illustrates the procedure. Path-summation prestack migration consists of stacking a prestack gather along different diffraction time trajectories (colour solid lines on the top of the gather) corresponding to different migration velocity models (different parameters α in formula (5)), where one of them (shown in red) coincides with the trajectory corresponding to the correct migration velocity. Vertical position of the migrated event corresponds to t_0 - for time migration (Fig. 1a), or to z - for depth migration (Fig. 1b). Note that in the depth migration case the summation is also applied in the time domain and that diffraction curves with different t_0 contribute to the same imaging z position (see Fig. 1b). Results of the path-summation migration are shown on the right hand side of each gather (Fig. 1a for time and Fig. 1b for depth). The weighting function $w(\alpha)$ plays extremely important role in our procedure. In this study it was chosen to be proportional to the flatness of the common image gathers calculated for each migrated sample and for each migration trajectory.

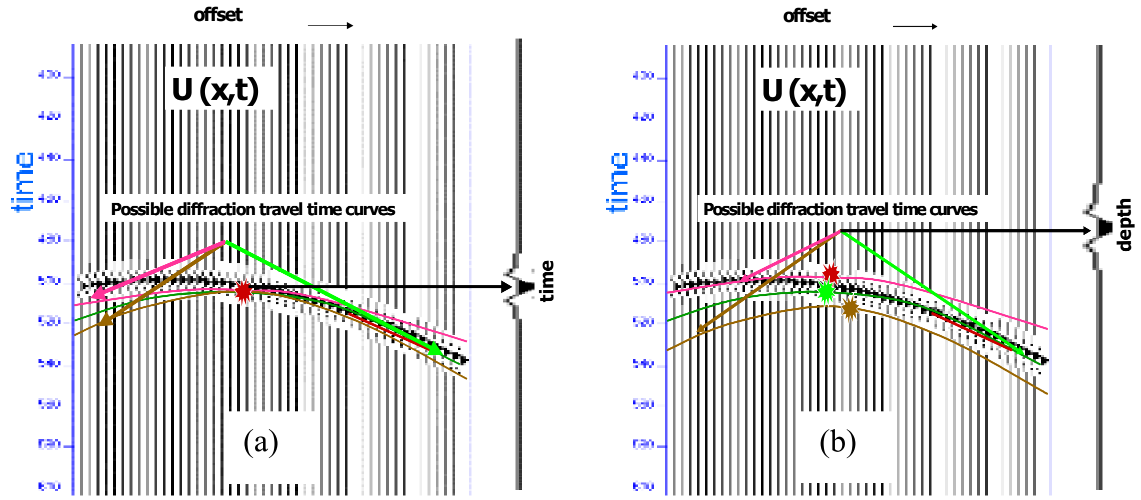

Figure 1. Scheme for path –summation time (a) and depth (b) migration.

In order to test the path-summation time migration procedure, a synthetic model from Claerbout (1995) is used. The model contains several features that challenge the migration performance: dipping beds, unconformity, complex structures and faults. Figure 2a shows the resulting section obtained by path-summation PSTM using migration velocity range between 1100 and 2800 m/sec. This result is compared to Figure 2b, which represents the result obtained by conventional PSTM using the correct velocity (constant and equal to 1500 m/sec).
The comparison clearly demonstrates the ability of the path-summation migration to accurately reproduce the correct time image without using any a priori information about migration velocity and without manual or automatic velocity analysis.

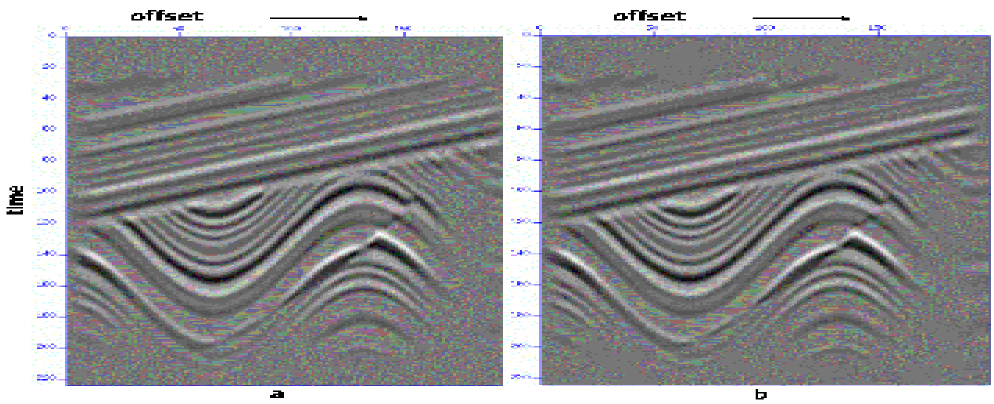

Figure 2: (a) Path-summation PSTM; (b) PSTM using the correct velocity.

In a similar way, the path-summation method is used for an "automatic" depth imaging without precise knowledge of the velocity model. Figure 3 illustrates a synthetic model which was used to test the depth imaging procedure. It consists of dipping and curved reflectors and contains strong lateral velocity variations. Figure 4 shows the resulting path-summation depth image, which was obtained by weighted summation over 250 different realizations of velocity model. Green's functions and common image gathers (CIG) for each velocity realization were calculated using an eikonal solver. For each imaging position $\vec{x}$ and velocity realization α, weighting function was calculated in the form:

$$w(\vec{x},\alpha) \sim \exp(-p(\vec{x},\alpha)) \qquad (6)$$

where p is a flatness index representing residual moveout (RMO) calculated for each sample of the CIGs. Figure 5 illustrates the flatness index and weighting function for two different velocity realizations: less probable (left) and more probable (right).

Conclusions

A heuristic development of a path-summation method to obtain approximated zero-offset or time/depth migrated images was presented. The convergence of this method depends on the range and sampling interval of the time trajectories of the path-summation parameter α . Our applications of the path-summation imaging indicate that it can accurately reproduce the main features of the subsurface without precise knowledge of the subsurface velocity. This study represents some ideas about how to choose a weighting function, sampling and number of velocity model realizations for path-summation. Obtaining reliable results, is still a matter of further research.

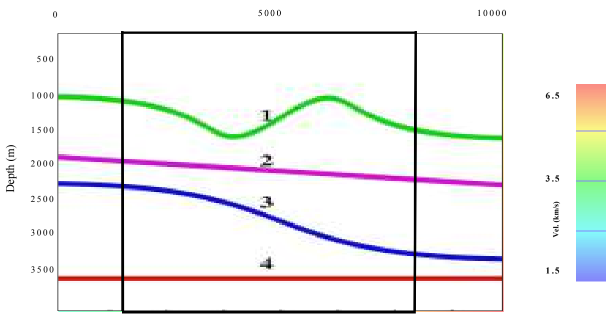

Figure 3: Synthetic model.

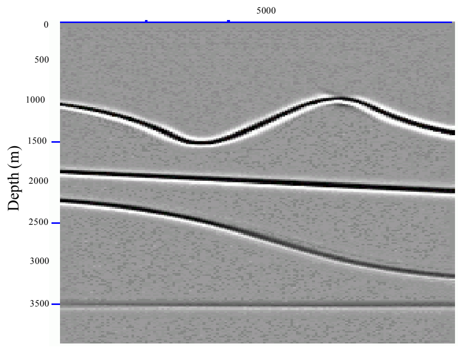

Figure 4: Path-summation PSDM.

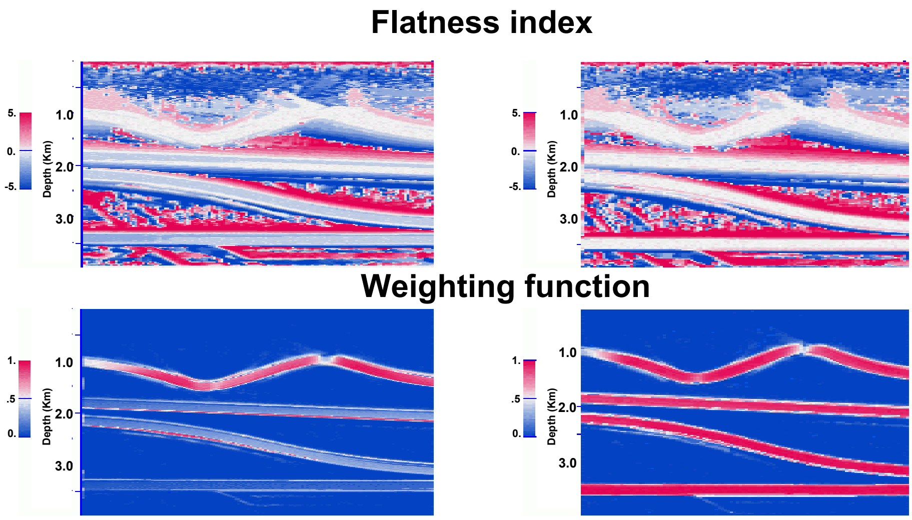

Figure 5: Flatness index and weighting function.

Acknowledgments

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